ELECTRICAL

GENERATION AND APPLICATION OF SENSITIVITY COEFFICIENTS

Prepared by

CONTROL SYSTEMS SENSITIVITIES GROUP

AUBURN UNIVERSITY

C. L. PHILLIPS, PROJECT LEADER

FINAL TECHNICAL REPORT

29 OCTOBER 1969 to 27 SEPTEMBER 1970

CONTRACT NAS8-21368
GEORGE C. MARSHALL SPACE FLIGHT CENTER
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
HUNTSVILLE, ALABAMA

ENGINEERING

ENGINEERING EXPERIMENT STATION
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FOREWORD

This document is the final technical summary for contract NAS8-21368. This contract was awarded to the Engineering Experiment Station, Auburn, Alabama, June 28, 1968, and was extended June 27, 1969 by the George C. Marshall Space Flight Center, National Aeronautics and Space Administration, Huntsville, Alabama.

SUMMARY

Methods for the generation of the first and second order sensitivity coefficients for systems whose input-output relationship is describable by a linear, ordinary, constant coefficient differential equation of the following type,

$$a_n \frac{d^n c}{dt^n} + a_{n-1} \frac{d^{n-1} c}{dt^{n-1}} + \ . \ . \ . + a_1 \frac{dc}{dt} + a_0 c =$$

$$b_n \frac{d^n r}{dt^n} + b_{n-1} \frac{d^{n-1} r}{dt^{n-1}} + \ . \ . \ . + b_1 \frac{dr}{dt} + b_0 r, \quad (1)$$

are presented in this report. The equation's coefficients may be functions of the system parameters and the order of numerator dynamics may be as great as the order of the denominator dynamics. It is shown that the first order sensitivity coefficients with respect to each parameter, pj, j=1,2,...,k, may be generated as linear combinations of the signals present in the system and one senstitivity model. The generation of the second order sensitivity coefficients with respect to each parameter pj may be accomplished with k+1 sensitivity models in addition to the system model.

The first and second order output sensitivities may be used for the purpose of generating the first and second order sensitivities of a class of cost functionals. The

cost sensitivities in turn are utilized for the purpose of determining parameter sets which yield a relative minimum in the cost functional.

An s-domain proof (Laplace transform) of the often noted symmetry and complete simultaneity property of the first order state sensitivities of a system in the companion canonic form [5] is given. This proof is extended to the second order state sensitivities. The implication of this result is that the second order state sensitivities with respect to any given number of system parameters may be generated utilizing two sensitivity models instead of k+1. Futhermore, the system need not be in the form of equation (1). Removal of this restriction means that one does not need the transfer function between input and output. The system may be simulated in any form desired.

LIST OF PERSONNAL

The following named staff members of Auburn University have actively participated on this project:

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C. E. Kulas - Instructor of Electrical Engineering

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I. INTRODUCTION

The purpose of this report is to present methods for generating the first and second order cross output sensitivity coefficients for systems whose input-output relationship is describable by a linear, ordinary, constant coefficient differential equation of the type

$$a_n \frac{d^n c}{dt^n} + a_{n-1} \frac{d^{n-1} c}{dt^{n-1}} + \ . \ . \ . + a_1 \frac{dc}{dt} + a_0 c =$$

$$b_n \frac{d^n r}{dt^n} + b_{n-1} \frac{d^{n-1} r}{dt^{n-1}} + \ . \ . \ . + b_1 \frac{dr}{dt} + b_0 r \ . \qquad \text{(I-1)}$$

The equation's coefficients may be functions of pj, for j=1,2,...,k system parameters and the order of numerator dynamics may be as great as the order of the denominator dynamics.

In this report a first order output sensitivity coefficient is the $\partial c(t)/\partial pj$ and a second order cross output sensitivity coefficient is the $\partial^2 c(t)/\partial pj \partial pi$. For all j=i it is called the second order output sensitivity coefficient. Since the sensitivity coefficients are by definition evaluated at the nominal values of pj, no special notation is used to indicate this. The class of cost functionals (PI) whose first and second order sensitivities are to be generated in this report are of the form

$$PI = \int_0^T F(c(t,\underline{p}))dt. \qquad (I-2)$$

In the above equation $c(t,\underline{p})$ is the system's output, $\underline{p}$ is the parameter set for which the PI sensitivities are to be generated and $F(c(t,\underline{p}))$ is a functional relationship of the output. The first and second order cost sensitivities are used for the purpose of minimizing the cost functional with respect to a set of system parameters for a given input.

II. THE SENSITIVITY MODELS

In this Chapter a procedure is given for the generation of the first, second and second order cross sensitivity coefficients. The second order cross sensitivity coefficient is defined as $\partial^2 c(t)/\partial pj \partial pi$ where $c(t)$ is the system's response to a given input and pj,pi are the system parameters. For all i=j the above partial is called the second order sensitivity coefficient.

First Order Sensitivity Model

The Laplace transform of equation (I-1) assuming zero initial conditions is

$$\frac{C}{R} = \frac{b_n s^n + b_{n-1} s^{n-1} + \ldots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0} = \frac{N}{D} \quad . \tag{II-1}$$

The partial of C with respect to a parameter pj is

$$\frac{\partial C}{\partial pj} = \frac{RD\partial pjN - RN\partial pjD}{D^2} = \frac{1}{a_n} \, \frac{R\partial pjN - C\partial pjD}{D/a_n} \quad , \tag{II-2}$$

where $\partial pj(\ )$ denotes $\partial(\ )/\partial pj$. It has been shown that the first order sensitivity coefficients may be obtained from the signals present at the nodes in Figure 1 [1,2]. That is,

$$\partial pjc(t) = \sum_{\ell=o}^{n} (M_\ell \partial pjb_\ell - S_\ell \partial pja_\ell) \tag{II-3}$$

for j=1,2,...,k. The familiar "Method of Sensitivity Points" which applies to systems whose input-output relationship is describable by a differential equation

$$\frac{d^n c}{dt^n} + a_n \frac{d^{n-1} c}{dt^{n-1}} + \ . \ . \ . + a_2 \frac{dc}{dt} + a_1 c = r, \qquad \text{(II-4)}$$

is shown in Figure 2 [3]. Clearly, Figure 2 is a special case of Figure 1.

<u>Second Order Sensitivity Model</u>

Let $\partial pjpi(\ )$ denote $\partial^2(\ )/\partial pj\partial pi$. The second order cross partial of C (second order partial for j=i) is

$$\partial pjpiC = \frac{DR\partial pjpiN-DC\partial pjpiD-D\partial piC\partial pjD-R\partial pjN\partial piD+C\partial pjD\partial piD}{D^2} .$$

From equation (II-2)

$$R\partial pjN = D\partial pjC + C\partial pjD \quad .$$

The above two equations yield

$$\partial pjpiC = \frac{1}{a_n} \ \frac{R\partial pjpiN-C\partial pjpiD-\partial piC\partial pjD-\partial pjC\partial piD}{D/a_n}. \qquad \text{(II-5)}$$

The system shown in Figure 1 is now represented as shown in Figure 3 with k additional sensitivity models whose inputs are the first order sensitivities as indicated. The second order cross sensitivities may be obtained from the summation of the signals present at the nodes in Figure 3 in accordance with the following equation,

$$\partial pjpiC = \sum_{\ell=o}^{n}(M_\ell \partial pjpib_\ell - S_\ell \partial pjpia_\ell - {}_iR_\ell \partial pja_\ell - {}_jR_\ell \partial pia_\ell), \qquad \text{(II-6)}$$

where i,j are elements of the number set 1,2,...,k and

${}_{i}R_{\ell}$ denotes node ℓ of the sensitivity model whose input is the ∂pic(t). Clearly, the total number of sensitivity models required for the generation of k second order sensitivity coefficients is k+1.

Discussion of the Given Techniques

The advantage of the application of the model shown in Figure 3 for the generation of the first and second order sensitivities is that the analog or digital simulation is strictly procedural and consequently requires only a small acquaintance with simulation methods. In particular it is readily adaptable to digital simulation utilizing the continuous system modeling procedures available in the present day digital computer. The disadvantage is that the transfer function between the point designated as input and the point designated as output must be available. This requirement in addition to the need of evaluating the partials of the transfer function's coefficients limits the practical application of the method to low order systems. The removal of these restrictions is discussed in Chapter V.

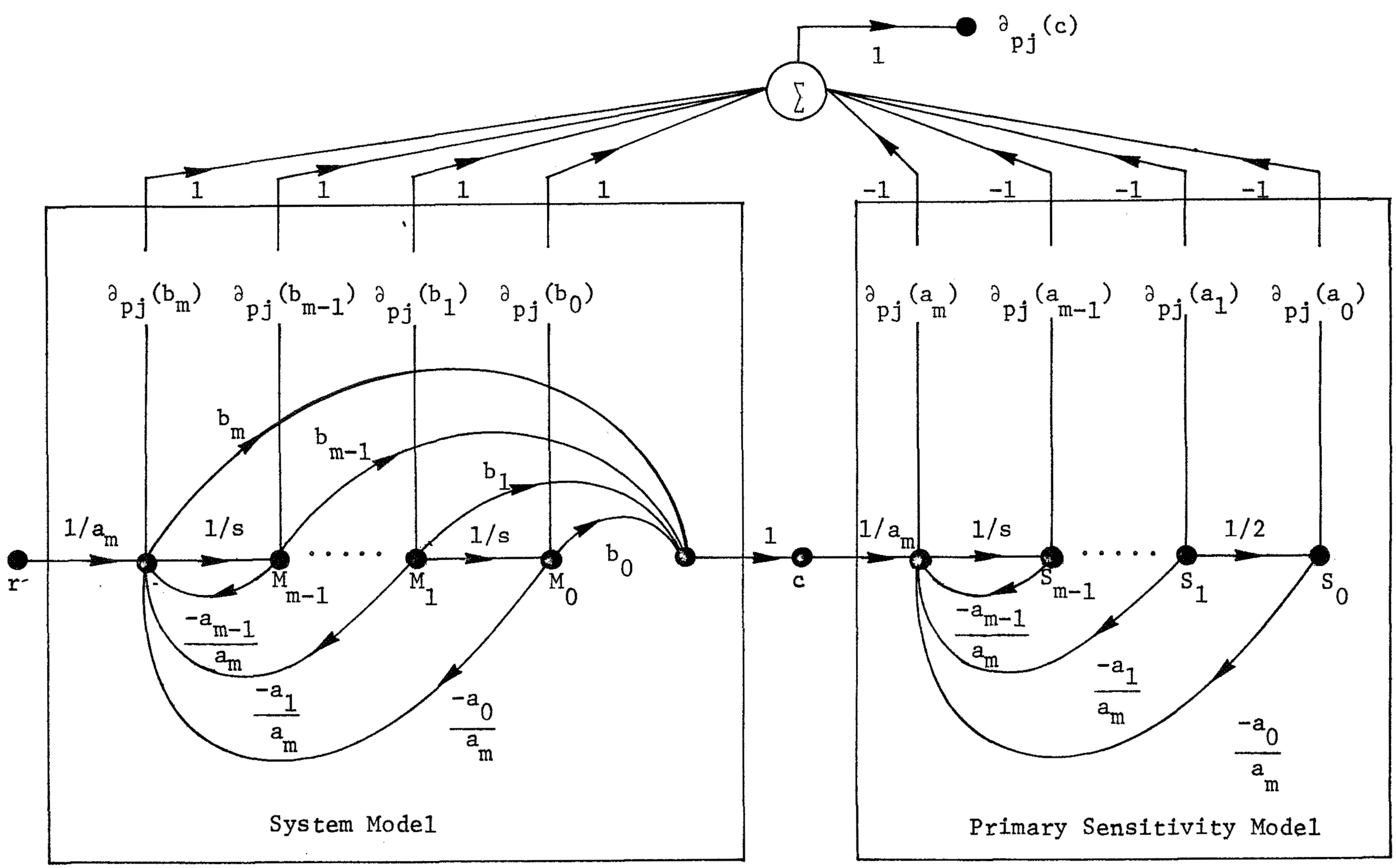

Figure 1. System and Sensitivity Model for First Order Sensitivity Coefficients

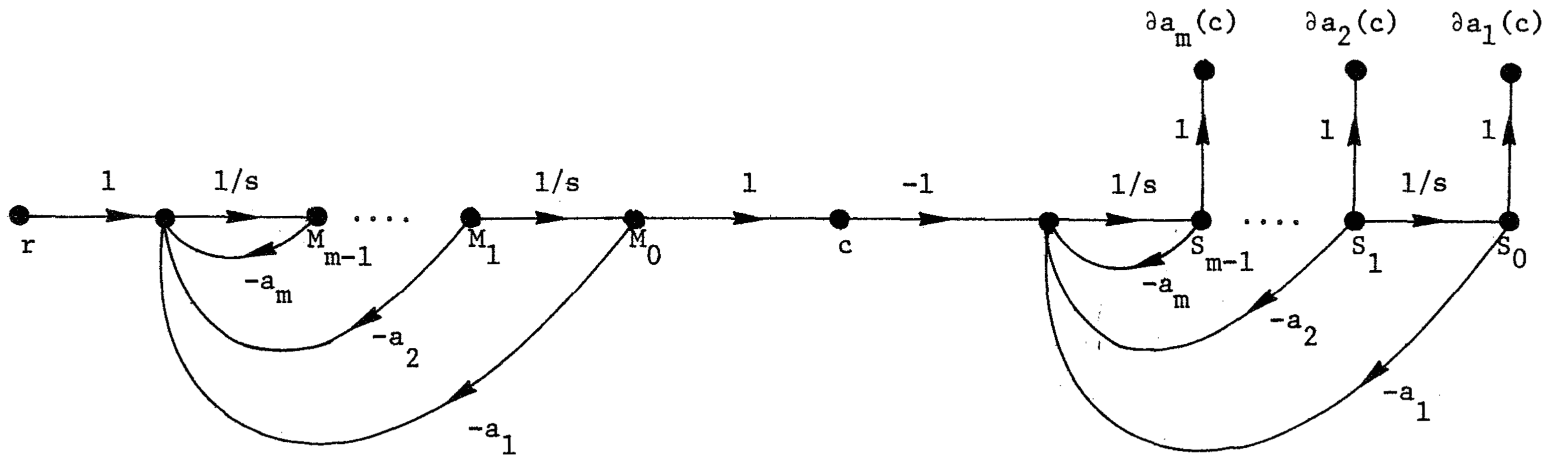

Figure 2. The "Method of Sensitivity Points".

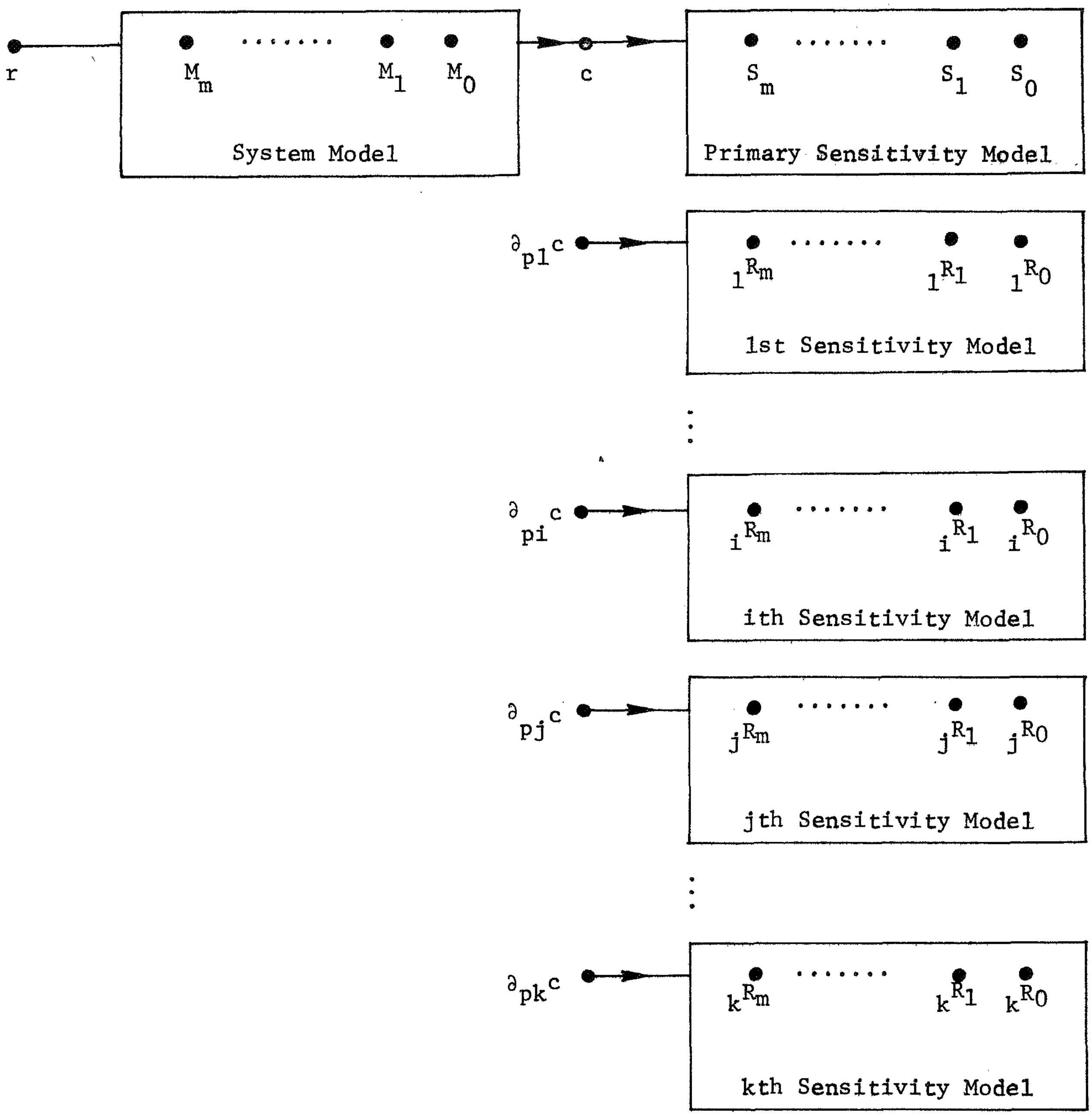

Figure 3. System and k+1 Sensitivity Models for Second Order Cross Sensitivities.

III. APPLICATION OF THE SENSITIVITY COEFFICIENTS

The first and second order output sensitivities may be used for the purpose of generating the first and second order sensitivities of a class of cost functionals. Of particular interest are cost functionals (PI) of the following type,

$$PI = \int_0^T F(c(t,\underline{p}))dt \ , \qquad \text{(III-1)}$$

where $c(t,\underline{p})$ is the response of the system to a given input, $\underline{p}$ is the parameter set pj for which the PI sensitivities are to be generated and $F(c(t,\underline{p})$ is a functional relationship of the response.

The first order PI sensitivity with respect to a parameter pj is

$$\frac{\partial PI}{\partial pj} = \int_0^T \frac{\partial F(c)}{\partial c} \frac{\partial c}{\partial pj} dt \ . \qquad \text{(III-2)}$$

The second order PI sensitivity with respect to pj is

$$\frac{\partial^2 PI}{\partial pj^2} = \int_0^T \frac{\partial F(c)}{\partial c} \frac{\partial^2 c}{\partial pj^2} + \frac{\partial^2 F(c)}{\partial c^2} \left(\frac{\partial c}{\partial pj}\right)^2 \ dt. \qquad \text{(III-3)}$$

The first and second order PI sensitivities may be generated from the signals available at the nodes in Figure 3, Chapter II. In particular, consider the rigid body dynamics of a launch vehicle [4],

$$\ddot{z} = K_3\phi + K_7\alpha + K_4\beta \ ,$$
$$\ddot{\phi} = -c_1\alpha - c_2\beta \ ,$$
$$\alpha = \alpha_W + \phi - \dot{z}/V \ ,$$

with the control law

$$\beta = a_o\phi + a_1\dot{\phi} + b_o\alpha \ . \qquad \text{(III-4)}$$

The first and second order PI sensitivities for

$$PI(z) = \int_o^T (z(t,a_o,a_1,b_o))^2 dt \qquad \text{(III-5)}$$

may be generated with respect to the control law gains for an arbitrary input α_W utilizing the model shown in Figure 3, Chapter II. For the inputs shown in Figure 4, the resulting sensitivities for sets of control law gains are given in Table 1.

The PI sensitivities may be used for the purpose of determining the parameter set which produces a relative minimum in the cost functional. The sign of the first order PI sensitivity shows the direction of change in the members of the parameter set which will produce a smaller cost at the end of the next simulation run. For example, twenty-six successive computer runs starting with a_o=6.0, a_1=2.0, b_o=2.0 and INPUT 1 produced a relative minimum of PI=0.027 and a parameter set of a_o=4.7, a_1=1.57, b_o=2.5. For each run the parameters were changed by 1% in accordance with the sign of the first order PI. The choice of 1% as an appropriate parameter change was made based on experience with this particular problem. The second order PI sensitivities may be used for the purpose of determining how large a change may be made in each of the parameters. For example, the approximation to the cost on each successive computer run with respect to parameter a_o is

$$PI(a_o + \Delta a_o) = PI(a_o) + \frac{\partial PI}{\partial a_o} \Delta a_o + \frac{\partial^2 PI}{\partial a_o{}^2} \frac{\Delta a_o{}^2}{2} . \quad \text{(III-6)}$$

From Table 1 and INPUT 1 for a_o=6.0, a_1=2.0, b_o=2.0, the $\partial^2 a_o PI$ is 2.93 and $\partial a_o PI$ is 10.49. For a parameter change of 1% the contribution of the second order term in (III-6) to the new cost is approximately 0.1% of the first order term. Clearly, for this particular computer run the magnitude of change in a_o can be considerably larger than 1% under the restriction that the first order approximation to the new cost is valid. Thus, the magnitude of the second order PI sensitivities may be used as a basis upon which the decision of how large a change may be made in each of the parameters for each successive run.

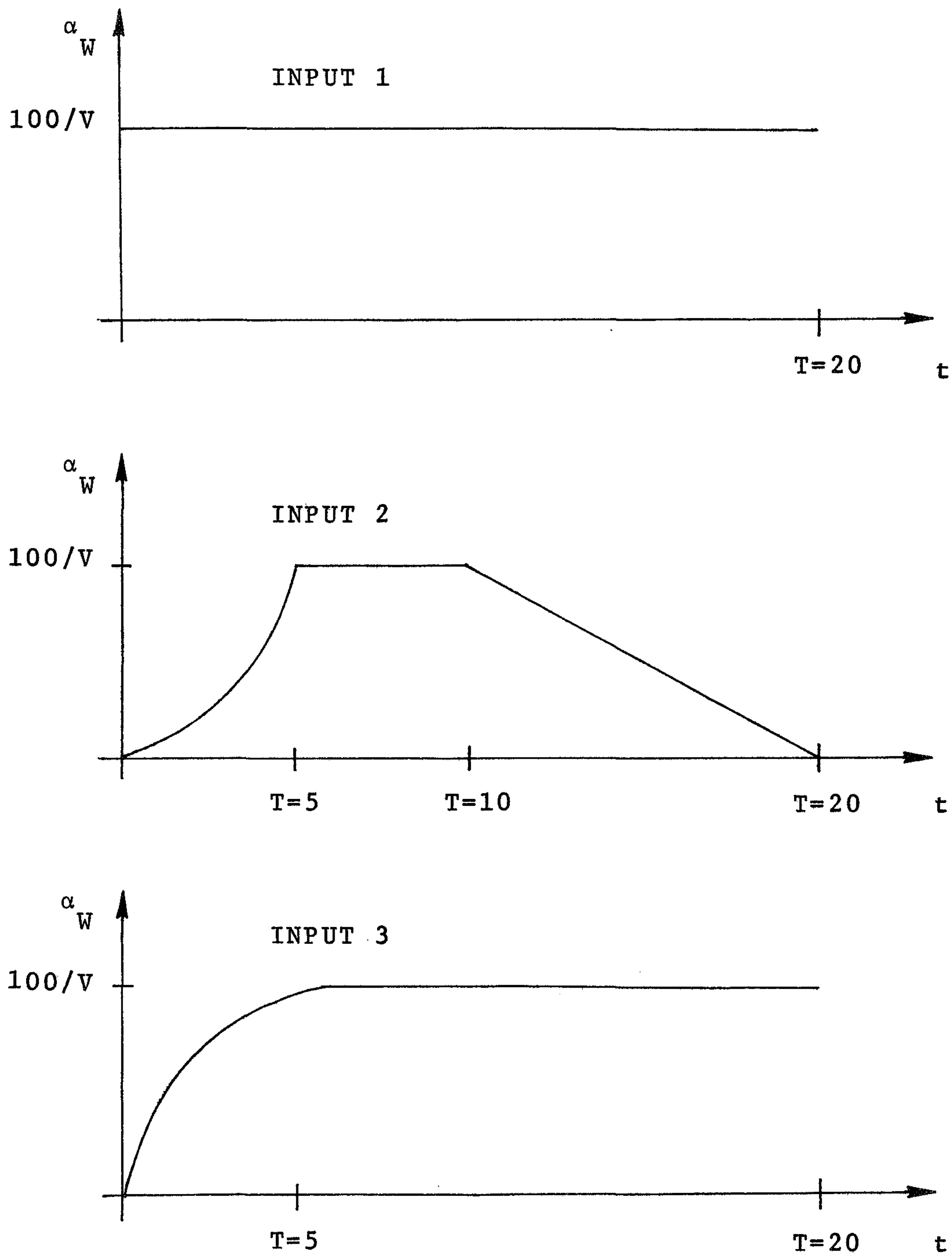

Figure 4. Inputs α_W for V=483.75 m/sec.

	$\partial a_o PI$	$\partial a_1 PI$	$\partial b_o PI$	$\partial a_o PI$	$\partial a_1 PI$	$\partial b_o PI$	PI
$a_o=6$ $a_1=2$ $b_o=2$							
INPUT 1	10.49	1.36	-0.36	2.93	0.05	35.70	18.78
INPUT 2	3.70	0.53	12.79	1.03	0.02	12.28	6.68
INPUT 3	6.46	0.92	-22.33	1.77	0.03	21.21	11.77
$a_o=2$ $a_1=2$ $b_o=6$							
INPUT 1	-134.22	-17.33	52.23	30.30	0.54	4.59	297.24
INPUT 2	-47.25	-6.61	18.55	10.64	0.23	1.64	104.90
INPUT 3	-81.28	-11.50	32.00	18.35	0.39	2.84	180.00
$a_o=2$ $a_1=2$ $b_o=2$							
INPUT 1	-49.93	-6.56	62.21	44.96	0.84	70.04	27.77
INPUT 2	-17.29	-2.48	21.75	15.60	0.36	24.86	9.6
INPUT 3	-29.44	-4.24	37.13	26.86	0.60	42.90	16.17

TABLE 1

IV. PROPERTIES OF THE STATE SENSITIVITIES

An s-domain proof of the noted total symmetry and complete simultaneity properties of the first order sensitivities of the states of a system whose input-output relationship is describable by the following type of differential equation,

$$\frac{d^n c}{dt^n} + a_n \frac{d^{n-1} c}{dt^{n-1}} + \ . \ . \ . + a_2 \frac{dc}{dt} + a_1 c = r \ , \qquad \text{(IV-1)}$$

is given [5]. It is further shown that the second order state sensitivities also possess the complete simultaneity property. Furthermore, it is shown that the second order state sensitivity matrix is such that

$$\alpha_{\beta,\rho} = \alpha_{\beta+1,\rho-2} \qquad \text{(IV-2)}$$

for $\beta=1,2,...,n-1$ and $\rho=3,4,...,n$. The implication of this is discussed in Chapter V.

First Order State Sensitivities

The Laplace transform of equation (IV-1) assuming zero initial conditions is

$$C = \frac{R}{s^n + a_n s^{n-1} + \ . \ . \ . + a_2 s + a_1} = \frac{R}{D} \ . \qquad \text{(IV-3)}$$

A signal flow graph of the above system with it's first order sensitivity model is shown in Figure 2, Chapter II. Let C(s), the transform of the output c(t) be denoted by $X_1(s)$; the transform of the signal at node M_1 be denoted by $X_2(s)$; . . . ; and the transform of the signal at node M_{n-1} be denoted by $X_n(s)$. From equation (IV-3) it follows that

$$\begin{aligned} \partial a_1 X_1 &= -s^o X_1/D, \\ \partial a_2 X_1 &= -s^1 X_1/D, \\ &\vdots \\ \partial a_{n-1} X_1 &= -s^{n-2} X_1/D, \\ \partial a_n X &= -s^{n-1} X_1/D. \end{aligned} \tag{IV-4}$$

Letting $\underline{X}^T$ denote $[X_1, X_2, \ldots, X_n]$ it further follows that

$$\begin{bmatrix} \partial a_1 \underline{X}^T \\ \partial a_2 \underline{X}^T \\ \partial a_3 \underline{X}^T \\ \vdots \\ \partial a_n \underline{X}^T \end{bmatrix} = \frac{-X_1}{D} \begin{bmatrix} s^o & s^1 & s^2 & \ldots & s^{n-1} \\ s^1 & s^2 & s^3 & \ldots & s^n \\ s^2 & s^3 & s^4 & \ldots & s^{n+1} \\ \vdots & & & & \\ s^{n-1} & s^n & s^{n+1} & \ldots & s^{2n-2} \end{bmatrix} \tag{IV-5}$$

since $X_2 = sX_1$, $X_3 = s^2 X_1, \ldots, X_n = s^{n-1} X_1$. The above sensitivity matrix shows that the first order state sensitivities possess the often noted total symmetry property [4]. The entries in (IV-5) for which the order of s is less than or equal to n are clearly available at the nodes of the sensitivity model in Figure 2, Chapter II. The first order state sensitivities associated with the entries in (IV-5) for which the order of s is greater than n may be formed from linear combinations of the signals at the nodes of Figure 2, Chapter II. Letting S_W^Z denote $\partial a_Z X_W$ it follows that

$$S_3^n = -s^{n+1}X_1/D = -X_1 s + a_n X_1 s^n + \dots + a_2 X_1 s^2 + a_1 X_1 s \ ,$$

$$S_3^n = -X_2 - a_n S_n^2 - \dots - a_2 S_2^2 - a_1 S_1^2 \ ,$$

$$S_4^n = -X_3 - a_n S_3^2 - \dots - a_2 S_3^2 - a_1 S_2^2 \ ,$$

$$\vdots$$

$$S_n^n = -X_{n-1} - a_n S_{n-1}^n - \dots - a_2 S_{n-1}^2 - a_1 S_{n-2}^2 \ ,$$

which can be written as

$$S_d^n = -X_{d-1} - \sum_{i=3}^{d-1} a_{n-(d-1-i)} S_i^n - \sum_{q=d-2}^{n} a_{q-(d-3)} S_q^2 \qquad \text{(IV-6)}$$

for d=4,5,...,n.

Second Order State Sensitivities

The second order partial of (IV-3) is

$$\partial a_1^2 X_1 = (-Ds^{o} \partial a_1 X_1 + s^{o} X_1)/D = 2s^{o} X_1/D^2 \ ,$$

$$\partial a_2^2 X_1 = \dots\dots\dots\dots = 2s^2 X_1/D^2 \ ,$$

$$\vdots$$

$$\partial a_n^2 X_1 = \dots\dots\dots\dots = 2s^{2n-2} X_1/D^2 . \qquad \text{(IV-7)}$$

From equations (IV-5) and (IV-7) it follows that

$$\begin{bmatrix} \partial a_1^2 \underline{X}^T \\ \partial a_2^2 \underline{X}^T \\ \partial a_3^2 \underline{X}^T \\ \cdot \\ \cdot \\ \cdot \\ \partial a_n^2 \underline{X}^T \end{bmatrix} = \frac{2X_1}{D^2} \begin{bmatrix} s^0 & s^1 & s^2 & \cdots & s^{n-1} \\ s^2 & s^3 & s^4 & \cdots & s^{n+1} \\ s^4 & s^5 & s^6 & \cdots & s^{n+3} \\ \cdot & & & & \\ \cdot & & & & \\ \cdot & & & & \\ s^{2n-2} & s^{2n-1} & s^{2n} & \cdots & s^{3n-3} \end{bmatrix} \qquad \text{(IV-8)}$$

The above second order state sensitivity matrix has the property that

$$\alpha_{\beta,\rho} = \alpha_{\beta+1,\rho-2} \qquad \text{(IV-9)}$$

for $\beta=1,2,\ldots,n-1$ and $\rho=3,4,\ldots,n$.

The entries in (IV-8) for which the order of s is less than or equal to n are available at the nodes of a second sensitivity model whose input is the output of the sensitivity model shown in Figure 2, Chapter II. The state sensitivities for which the order of s is greater than n may be formed through linear combinations of the available first and second order state sensitivities as is shown below.

Let Q_W^Z denote $\partial a_Z^2 X_W$. It follows that

$$\partial a_2^2 X_n = \frac{2X_1 s^{n+1}}{D^2} = \frac{2X_1 s}{D} + \frac{-2a_n X_1 s^n - 2a_{n-1} X_1 s^{n-1} - \,..\, -2a_1 X_1 s}{D^2} ,$$

$$Q_n^2 = -2S_1^2 - a_n Q_{n-1}^2 - a_{n-1} Q_{n-2}^2 - \,..\, -a_2 Q_1^2 - a_1 Q_2^1 ,$$

$$Q_{n-1}^3 = -2S_1^3 - a_n Q_n^2 - a_{n-1} Q_{n-1}^2 - \,..\, -a_2 Q_2^2 - a_1 Q_1^2 ,$$

$$Q_n^3 = -2S_2^3 - a_n Q_{n-1}^3 - a_{n-1} Q_n^2 - \,..\, -a_2 Q_3^2 - a_1 Q_3^2 ,$$

$$\vdots$$

$$Q_{n-1}^n = -2S_{n-2}^n - a_n Q_n^{n-1} - a_{n-1} Q_{n-1}^{n-1} - \,..\, -a_2 Q_2^{n-1} - a_1 Q_1^{n-1} ,$$

$$Q_n^n = -2S_{n-1}^n - a_n Q_{n-1}^n - a_{n-1} Q_n^{n-1} - \,..\, -a_2 Q_3^{n-1} - a_1 Q_2^{n-1} .$$

V. CONCLUSIONS

It has been shown that for single-input single-output Laplace transformable systems, that the first and second order cross output sensitivities, in addition to the systems response to a given input, may be generated through the utilization of the system model and k+1 sensitivity models where k is the number of parameters whose output sensitivities are to be generated. The system may have numerator dynamics of order equal to or less than the order of the denominator dynamics and the coefficients of the system may be functions of the k parameters. The advantage of the given method is that once the order of the system is known the structure of the k+1 sensitivity models is specified and consequently may be simulated either on an analog or digital computer. The sensitivities in turn are linear combinations of the signals available at the nodes in the system and sensitivity models. The disadvantage is that the system's transfer function must be known and that the first order, second order and second order cross partials of the transfer function's coefficients must be evaluated at the nominal values of the parameter set for which the sensitivities are desired.

The first and second order sensitivities of a class of cost functionals may be obtained from the output sensitivities.

The availability of the first order cost sensitivities allows one to seek out parameter sets which reult in relative minimums of the cost. The availability of the second order sensitivities allows one to make an intelligent decision as to how large of a change may be taken for each of the parameters in each successive computer run in the optimization scheme. Application of the second order cost sensitivities may result in a savings of computer run time. Experience with the problem of minimizing drift has shown that for parameter sets (control law gains a_o, a_1, b_o) which are "far" from the set which produces "drift minimum" large changes may be taken in the parameter set in each computer run until one reaches the neighborhood of the optimal parameter combination. The magnitude of the available second order cost sensitivities reflects this and consequently have been successfully used in the optimization procedure. For example, if the changes in the parameters is restricted to 1% of the nominal values, excessive computer run time is required in reaching the optimal values. However, if the percentage change is varied in accordance with the magnitude of the second order cost sensitivities the computer run time can be considerably reduced.

The primary disadvantage of the procedure given in this report is that the system's transfer function must be available and that the system must be simulated in the form of Figure 1,Chapter II. It has been shown [5] that this restriction can be removed. The basis for the removal of this restriction is the proof of the complete simultaneity property of a system in the companion canonic form (the form of Equation IV-1). Section IV of this report shows that the second order state sensitivities of the companion canonic form also possesses the complete simulataneity property. Consequently the work shown in Reference [5] can be extended to the second order case. The removal of the restriction on the form of the system greatly complicates the digital simulation procedure. The second order case in turn would be even more complicated. However, the freedom of simulating the system in any form desirable warrants research into the extension of the method given in Reference [5].

Much of the material presented in Chapter II of this report appears to be extendable to the case of unity negative feedback systems with one nonlinearity. Research into this area should be most productive. The availability of the sensitivities in the nonlinear case could result in techniques for optimization where the object is not to find the optimal controller but to adjust parameters in a given controller which produces a minimum under this constraint. That is to say;

determine controller gains which realize a given objective utilizing a controller structure that is known to give satisfactory results but not the optimal result.

Another problem which warrants research is the generation of the output sensitivities in the case where the order of the assumed model may change. For example consider Figure 1, Chapter II; the coefficient a_n which is a function of the system parameters may be zero for a given parameter set. Since in analysis one often assumes certain parameters may be neglected which in turn result in the reduction of the system order, sensitivities of the system output with respect to a change in system order would be of significance in verifying the validity of such an assumption.

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